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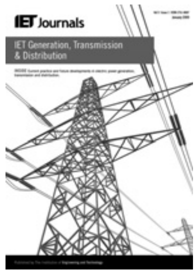

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Numerical observability method for optimal phasor measurement units placement using recursive Tabu search method

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Abstract: Phasor measurement units (PMUs) are essential tools for monitoring, protection and control of power systems. The optimal PMU placement (OPP) problem refers to the determination of the minimal number of PMUs and their corresponding locations in order to achieve full network observability. This paper introduces a recursive Tabu search (RTS) method to solve the OPP problem. More specifically, the traditional Tabu search (TS) metaheuristic algorithm is executed multiple times, while in the initialisation of each TS the best solution found from all previous executions is used. The proposed RTS is found to be the best among three alternative TS initialisation schemes, in regard to the impact on the success rate of the algorithm. A numerical method is proposed for checking network observability, unlike most existing metaheuristic OPP methods, which are based on topological observability methods. The proposed RTS method is tested on the IEEE 14, 30, 57 and 118-bus test systems, on the New England 39-bus test system and on the 2383-bus power system. The obtained results are compared with other reported PMU placement methods. The simulation results show that the proposed RTS method finds the minimum number of PMUs, unlike earlier methods which may find either the same or even higher number of PMUs.

1 Introduction

Phasor measurement unit (PMU) is a power system device capable of measuring the synchronised voltage and current phasor in a power system. Synchronisation among phasor measurements is accomplished by sampling of voltage and current waveforms with the same time-stamp, using the satellite-based global positioning system (GPS) [1]. Calculation of real-time synchronised measurements makes PMUs one of the most important measuring devices in monitoring, control and protection of power systems. An interesting application of PMUs is in state estimation (SE), which is an essential function in energy management systems (EMS), providing the voltage phasors at all network nodes.

The ability to perform SE depends on the measurement system observability [2]: if there are sufficient and well-distributed measurements throughout the network to uniquely estimate the states of a power system, the system is said to be observable. If the system is unobservable, observable islands will be identified and a minimal set of measurements that make the entire network observable will be provided. Observability analysis methods can be classified as numerical, topological and hybrid. The numerical methods are based on whether the measurement gain, Jacobian or Gram matrix is of full rank, whereas the topological methods rely on whether a spanning tree of full rank can be constructed. An iterative numerical procedure, using the triangular factors of the gain matrix, is proposed in [3, 4]. A direct (non-iterative) numerical algorithm for observability analysis and measurement placement, also based on the factors of the gain matrix, is presented in [5, 6], respectively. In [7], the proposed procedure is based on the Jacobian matrix instead of the gain matrix. A robust observability checking algorithm based on Gaussian elimination and binary arithmetic is suggested in [8]. A unified numerical algorithm for observability analysis and restoration is provided in [9]. A numerical technique based on the solution of a non-linear integer programming problem, which permits the determination of the minimum measurement set and ensures observability even if any k meters fail, is presented in [10]. An observability analysis technique based on orthogonal Givens rotations is introduced in [11]. By using the triangular factors of the Gram matrix associated with the Jacobian matrix, a non-iterative numerical method is proposed in [12]. Direct methods [13, 14] for observability analysis and restoration rely on the triangular factorisation of a gain and a Gram matrix, respectively, associated with a reduced order Jacobian, for systems comprising conventional and phasor measurements. An efficient iterative numerical algorithm for measurement placement is developed in [15]. Binary integer programming is implemented as part of the algorithm to optimise selection of measurements. In [16], a numerical method is suggested to incorporate phasor measurements as well as voltage magnitude measurements in observability analysis.

In [17], the fundamentals of a topological algorithm based on building a spanning tree of full rank are presented. In [18, 19], the procedure in [17] is extended to obtain the largest observable subnetworks of an unobservable network and the measurements for placement, respectively. Reference [20] identifies an observable spanning tree using an algorithm based on matroid intersections. Finally, an algorithm based on building a maximal forest of full rank is presented in [21]. Note that the topological techniques involve a combinatorial computational complexity, whereas the numerical techniques do not. Hybrid methods [22–25] combine efficiently both the numerical and topological techniques. Flow measurements are used to build topologically the flow islands that, in turn, are used to construct a reduced network. To build a matrix characterising the reduced network, only boundary nodes and injections at flow islands are considered. A numerical procedure is then used to process the matrix obtained and to check and restore observability.

As the PMUs are increasingly being deployed by electric utilities worldwide, although their cost remains high, the optimal PMU placement (OPP) problem has concentrated a great research interest [26]. The OPP problem concerns the minimisation of the number of installed PMUs in order to achieve full network observability. Different methodologies have been implemented to solve the OPP problem. They can be classified into two broad categories: (i) mathematical and (ii) heuristic algorithms [26].

An integer programming-based formulation for the solution of OPP problem is presented in [27], considering PMU measurements as well as conventional measurements (zero injections or power injections). In [28], the line outage or PMU loss contingency conditions, with or without the existence of zero injections, are considered separately or simultaneously in a linear programming model. The communication constraints can be also added as measurement limitations in the model. An equivalent integer linear programming method for the exhaustive search-based PMU placement is proposed in [29]. Additional constraints for observability preservation, following single PMU or line outages, can easily be implemented in the model. In [30], simulated annealing (SA) is used to solve the OPP problem. Three different approaches are proposed in [31]: (i) a modified simulated annealing (MSA) algorithm that modifies the settings of initial temperature and cooling procedure, (ii) a direct combination (DC) method that makes use of a heuristic rule to select the most effective set in the observability sense and (iii) a Tabu search (TS) method that uses the same heuristic rule as DC to reduce the searching space effectively. In [32], a topological method based on the augment incidence matrix and TS is proposed to solve the combinatorial optimisation problem and a priority list based on heuristic rule is embedded to accelerate optimisation. Several methods based on genetic algorithm principles are also proposed to solve the OPP [33–35].

The TS algorithm is an advanced metaheuristic optimisation method, which has already proven very efficient in solving complex power system optimisation problems [36], including the OPP [32]. The TS method differs from other optimisation techniques in the use of memory, which is crucial for the successful implementation of TS. As the TS algorithm traverses the solution space, it stores relevant findings in short-term and long-term memories, which are then subsequently used to redirect search and modify the local search algorithms that form part of TS metaheuristic [36]. In line with the recent findings, according to which one of the areas for future research in OPP is the development of advanced optimisation methods [26], this paper introduces two different approaches based on the metaheuristic TS algorithm to solve the OPP problem. In both approaches, the network observability is checked by a numerical observability analysis method [22], which guarantees reliable and fast results. In the first approach, called multiple Tabu search (MTS), TS is executed multiple times and the initial solution is computed by the greedy algorithm. Two different greedy algorithm initialisation schemes are proposed for the MTS method. The first initialisation scheme prohibits PMU placement at zero injection nodes, whereas the second scheme permits that placement. In the second approach, called recursive Tabu search (RTS), TS is executed recursively using as initial solution the best solution found in the previous executions. The proposed methods provide promising results that are validated using the IEEE standard test systems (14, 30, 57 and 118-bus), the New England 39 (NE 39)-bus test system and a 2383-bus large-scale power system by considering the effect of zero injection buses. The contributions of the paper are summarised below:

1. To the best of the authors' knowledge [26], this is the first time that a numerical observability analysis method is combined with a metaheuristic technique (such as TS, genetic algorithm, SA, differential evolution, particle swarm optimisation, etc.) to solve the OPP problem.
2. A new numerical observability analysis method [22], dedicated to PMU measured systems, is applied for the first time in combination with a TS method to solve the OPP problem.
3. Two different methods, namely RTS and MTS, are introduced for solving the OPP.
4. Three different TS initialisation schemes are proposed and their impact on the success rate of the algorithm is investigated.

This paper is organised as follows: Section 2 presents the numerical observability method used by the proposed MTS and RTS methods. Section 3 formulates the OPP problem and presents the proposed MTS and RTS methods. Section 4 presents and discusses the simulation results of the proposed methods. Conclusions are drawn in Section 5.

2 SE and observability analysis

Observability analysis is referred to the ability to estimate the power system state for a given set of measurements. The geographical distribution of measurements throughout the network is essential to solve the power system state estimation [3]. The numerical methods involve matrix analysis, whereas the topological approaches are based on graph theory to determine the network's observability.

In this paper, the numerical approach of [22] is used for observability checking. Considering an N-bus power system, the equations of the SE model are

$$\boldsymbol{z} = \boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{e} \quad (1)$$

$$\text{s.t. } \boldsymbol{c}(\boldsymbol{x}) = 0 \quad (2)$$

where $\boldsymbol{z}$ is the $m \times 1$ measurement vector, $\boldsymbol{x}$ is the $n \times 1$ state vector containing the bus voltage phasors, $\boldsymbol{h}(\cdot)$ is the $m \times 1$ measurement function vector, $\boldsymbol{e}$ is the $m \times 1$ error vector

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normally distributed with $E(e)=0$, $\boldsymbol{c}(\cdot)$ is the $l \times 1$ vector of functions to model zero injections as equality constraints, m is the number of measurements and $n=2N$ is the number of states. The measurement vector $\boldsymbol{z}$ is assumed to comprise only phasor measurements. The weighted least squares (WLS) method is used to minimise the following objective function

$$\boldsymbol{J}(\boldsymbol{x}) = (\boldsymbol{z}-\boldsymbol{h}(\boldsymbol{x}))^{\mathrm{T}}\boldsymbol{R}^{-1}(\boldsymbol{z}-\boldsymbol{h}(\boldsymbol{x})) \quad (3)$$

$$\text{s.t. } \boldsymbol{c}(\boldsymbol{x}) = 0 \quad (4)$$

where $\boldsymbol{R}=E(\boldsymbol{ee}^{\mathrm{T}})$ is the diagonal covariance matrix of measurement errors.

The estimated state $\hat{x}$ is obtained by iteratively solving the following system of linear equations [37]

$$\begin{pmatrix} \boldsymbol{G}(\boldsymbol{x}^k) & \boldsymbol{C}^{\mathrm{T}}(\boldsymbol{x}^k) \\ \boldsymbol{C}(\boldsymbol{x}^k) & 0 \end{pmatrix}\begin{pmatrix} \boldsymbol{x}^{k+1}-\boldsymbol{x}^k \\ \boldsymbol{\lambda}^{k+1} \end{pmatrix} = \begin{pmatrix} \boldsymbol{H}^{\mathrm{T}}(\boldsymbol{x}^k)\boldsymbol{R}^{-1}(\boldsymbol{z}-\boldsymbol{h}(\boldsymbol{x}^k)) \\ -\boldsymbol{c}(\boldsymbol{x}^k) \end{pmatrix} \quad (5)$$

where k is the iteration index, $\boldsymbol{x}^k$ is the solution vector at iteration k, $\boldsymbol{\lambda}^k$ are the Lagrange multipliers at iteration k, $\boldsymbol{H}(\boldsymbol{x})=\partial\boldsymbol{h}/\partial\boldsymbol{x}$ and $\boldsymbol{C}(\boldsymbol{x})=\partial\boldsymbol{c}/\partial\boldsymbol{x}$ are Jacobian matrices and $\boldsymbol{G}(\boldsymbol{x})=\boldsymbol{H}^{\mathrm{T}}(\boldsymbol{x})\boldsymbol{R}^{-1}\boldsymbol{H}(\boldsymbol{x})$ is the gain matrix. Iterations start at an initial estimate and are going on until the maximum state variable difference becomes less than a given threshold. Note that if the state vector is expressed in rectangular form, (5) becomes linear and the solution is directly obtained in one iteration.

A system is observable if the coefficient matrix $\mathbf{F} = \begin{pmatrix} \boldsymbol{G} & \boldsymbol{C}^{\mathrm{T}} \\ \boldsymbol{C} & 0 \end{pmatrix}$ of (5) has a full rank [18]

$$\text{rank}\begin{pmatrix} \boldsymbol{H}^{\mathrm{T}}\boldsymbol{H} & \boldsymbol{C}^{\mathrm{T}} \\ \boldsymbol{C} & 0 \end{pmatrix} = \text{rank}\begin{pmatrix} \boldsymbol{H} \\ \boldsymbol{C} \end{pmatrix} = n \quad (6)$$

$$\text{nullity}\begin{pmatrix} \boldsymbol{H}^{\mathrm{T}}\boldsymbol{H} & \boldsymbol{C}^{\mathrm{T}} \\ \boldsymbol{C} & 0 \end{pmatrix} = \text{nullity}\begin{pmatrix} \boldsymbol{H} \\ \boldsymbol{C} \end{pmatrix} = 0 \quad (7)$$

The observability analysis does not depend on the actual state of the system or the branch parameters, making it possible to use simplified linear ('DC') equations without loss of generality [3]. The equations can be further simplified by setting $R=0$ and $X=1$ for all branches. The linearised Jacobian matrices $\boldsymbol{H}$ and $\boldsymbol{C}$, referring to phasor measurements and zero injections, respectively, can be written either in polar (δ_i, V_i) or rectangular (E_i, F_i) coordinates as follows (see (8) and (9))

where δ_i^{meas}, V_i^{meas} are the measured voltage phase angle and magnitude at bus i, $I_{ij,\mathrm{r}}^{\text{meas}}$, $I_{ij,\mathrm{i}}^{\text{meas}}$ are the real and imaginary parts of the measured current phasor $\tilde{I}_{ij}$ at branch i–j, $I_{i,\mathrm{r}}^{\text{zero}}$, $I_{i,\mathrm{i}}^{\text{zero}}$ are the real and imaginary parts of the injected current phasor at zero injection bus i, and $\{j, k, \ldots\}$ and l_i are the set and number of the buses connected to bus i, respectively.

The power system will be observable iff [22]

$$\text{rank}\begin{pmatrix} \boldsymbol{H}^{\mathrm{T}}\boldsymbol{H} & \boldsymbol{C}^{\mathrm{T}} \\ \boldsymbol{C} & 0 \end{pmatrix} = \text{rank}\begin{pmatrix} \boldsymbol{H} \\ \boldsymbol{C} \end{pmatrix} = n \quad (10)$$

$$\text{nullity}\begin{pmatrix} \boldsymbol{H}^{\mathrm{T}}\boldsymbol{H} & \boldsymbol{C}^{\mathrm{T}} \\ \boldsymbol{C} & 0 \end{pmatrix} = \text{nullity}\begin{pmatrix} \boldsymbol{H} \\ \boldsymbol{C} \end{pmatrix} = n - \text{rank}\begin{pmatrix} \boldsymbol{H} \\ \boldsymbol{C} \end{pmatrix} = 0 \quad (11)$$

3 OPP problem formulation and proposed solution methods

3.1 OPP problem formulation

The OPP problem concerns the determination of the minimum number of PMUs, n_{PMU}, and the optimal location set, $S(n_{\text{PMU}})$, of the n_{PMU} PMUs ensuring full observability of the power system and maximum measurement redundancy, $R(n_{\text{PMU}}, S(n_{\text{PMU}}))$. It can be formulated as follows

$$\min_{n_{\text{PMU}}}\{\max R(n_{\text{PMU}}, S(n_{\text{PMU}}))\} \quad (12)$$

$$\text{s.t. Observability}(n_{\text{PMU}}, S(n_{\text{PMU}})) = 1 \quad (13)$$

$$\boldsymbol{H} = \left(\begin{array}{ccccccc|ccccccc} \cdots & \delta_i(E_i) & \cdots & \delta_j(E_j) & \cdots & \delta_k(E_k) & \cdots & \cdots & V_i(F_i) & \cdots & V_j(F_j) & \cdots & V_k(F_k) & \cdots \\ & 1 & & & & & & & & & & & & \\ & & & & & & & & 1 & & & & & \\ & 1 & & -1 & & & & & -1 & & 1 & & & \\ & 1 & & -1 & & & & & -1 & & 1 & & & \end{array}\right) \begin{array}{c} \vdots \\ \delta_i^{\text{meas}} \\ V_i^{\text{meas}} \\ \vdots \\ I_{ij,\mathrm{r}}^{\text{meas}} \\ I_{ij,\mathrm{i}}^{\text{meas}} \\ \vdots \end{array} \quad (8)$$

$$\boldsymbol{C} = \left(\begin{array}{ccccccc|ccccccc} \cdots & \delta_i(E_i) & \cdots & \delta_j(E_j) & \cdots & \delta_k(E_k) & \cdots & \cdots & V_i(F_i) & \cdots & V_j(F_j) & \cdots & V_k(F_k) & \cdots \\ & l_i & & -1 & & -1 & & & -l_i & & 1 & & 1 & \\ & l_i & & -1 & & -1 & & & -l_i & & 1 & & 1 & \end{array}\right) \begin{array}{c} \vdots \\ I_{i,\mathrm{r}}^{\text{zero}} \\ I_{i,\mathrm{i}}^{\text{zero}} \\ \vdots \end{array} \quad (9)$$

where Observability $(n_{PMU}, S(n_{PMU})) = 1$ is the observability logical function, which equals to '0' if the system is not observable or '1' if the system is observable.

3.2 Proposed OPP solution methods

This paper proposes an improved TS method for the solution of the OPP problem. The flowchart of the proposed method is shown in Fig. 1. The following subsections provide details about the proposed algorithm. The improved TS method is the basis for the proposed multiple and RTS OPP algorithm, introduced in Section 3.2.5.

3.2.1 TS overview: TS algorithm is a memory-based metaheuristic approach, which is used to solve large-scale combinatorial optimisation problems. TS was introduced

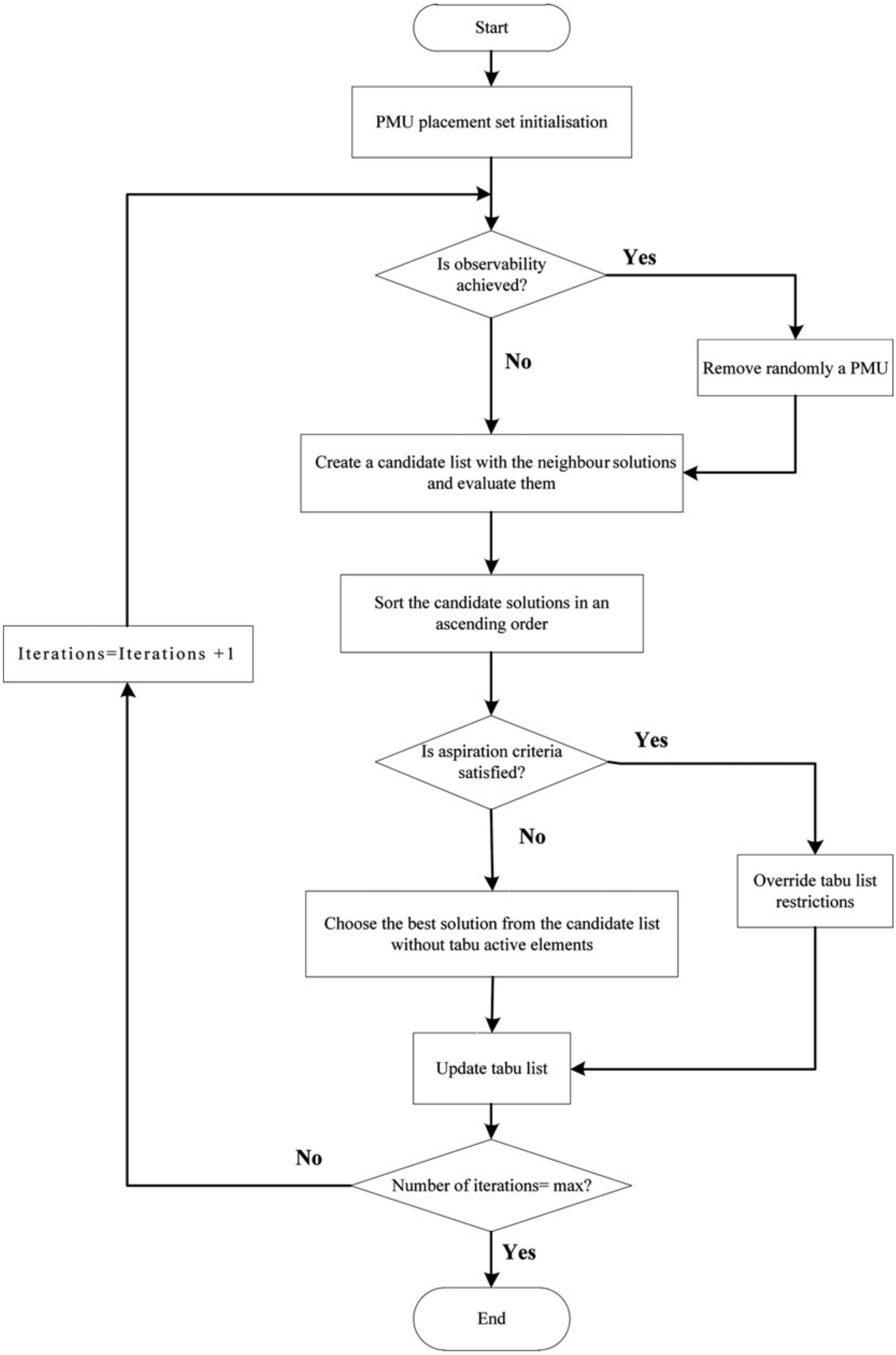

Fig. 1 *Flowchart of the proposed TS algorithm for the solution of the OPP problem*

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and established by Glover and it has a wide range of applications [38].

Starting from an initial solution x_0 generated by a heuristic algorithm, TS iteratively explores the whole neighbourhood $N(x)$ of the current solution x by defined means of movements and it evaluates and classifies the elements of $N(x)$ with a determined objective function. Then, in case of minimisation problems, the algorithm chooses the neighbour solution x' with the smallest value $f(x')$, regardless of being worse than the value of current solution $f(x)$.

In order to avoid the entrapment in a local optimum and the occurrence of cycling, each time a movement is done; it is stored in a Tabu list with a specific Tabu length (TL). Tabu list contains the most recent movements and it forbids their usage for TL iterations. Thus, the possibility of revisiting the TL − 1 last solutions is eliminated. An aspiration criterion is used to override the restrictions of the Tabu list. As a result, if a movement yields a solution x' whose value is smaller than the value of the best solution obtained so far, x^*, then this movement loses its Tabu status.

The algorithm terminates when a stopping criterion, such as a pre-specified number of iterations, is satisfied.

3.2.2 PMU placement initialisation: The greedy algorithm [39] is used to generate an initial solution of the OPP problem. The greedy algorithm is characterised by lack of sophistication, which makes it ideal for the generation of solutions without great computational effort. The basic steps of the greedy PMU placement initialisation algorithm are as follows:

Step 1: Form the incidence matrix $\boldsymbol{A}$ as follows [27]

$$\mathbf{A}_{k,m} = \begin{cases} 1, & \text{if } k = m, \text{ or } k \text{ and } m \text{ are connected} \\ 0, & \text{otherwise} \end{cases}$$

Step 2: Place a PMU at the bus with the maximum number of adjacent unobservable buses. If there are more than one buses having the same number of adjacent unobservable buses, then a PMU is randomly placed at one of them. The greedy PMU placement algorithm can be implemented either by prohibiting or not the placement of PMUs at zero injection nodes.
Step 3: Update the PMU set and the set of observable buses. When a PMU is placed at a bus, that bus along with its adjacent buses become observable.
Step 4: Check the observability with the numerical method of Section 2. If the system is observable, then the procedure is terminated, else proceed to Step 2.

It should be noted that the above PMU placement initialisation methodology provides PMU locations that ensure full network observability, although the placement will not be optimal.

3.2.3 Evaluation function: The following objective function is proposed for the evaluation of the candidate TS solutions

$$\min\{Q(n_{\text{PMU}}, S(n_{\text{PMU}}))\} = \min\left\{w_1 \cdot \text{nullity}\begin{pmatrix} \boldsymbol{H} \\ \boldsymbol{C} \end{pmatrix} + w_2 \cdot n_{\text{PMU}} - w_3 \cdot R(n_{\text{PMU}}, S(n_{\text{PMU}}))\right\} \tag{14}$$

where w_1 = number of buses, $w_2 = 1.0$ and $w_3 = 0.1$. As can be observed, weight w_1 depends on the network size, whereas weights w_2 and w_3 have a fixed value, with w_2 being one order of magnitude larger than w_3. The reason for selecting $w_1 > w_2 > w_3$ is to satisfy first the network observability, then the number of PMUs, and last the measurement redundancy. The above selection of values for w_1, w_2 and w_3 provides very good results for the OPP problem, as shown in Section 4. In (14), $\text{nullity}\begin{pmatrix} \boldsymbol{H} \\ \boldsymbol{C} \end{pmatrix}$ is the rank deficiency of the Jacobian matrix $\begin{pmatrix} \boldsymbol{H} \\ \boldsymbol{C} \end{pmatrix}$, associated with phasor measurements and zero injections, and is calculated as shown in Section 2. Moreover, in (14), n_{PMU} is the number of PMUs and $R(n_{\text{PMU}}, S(n_{\text{PMU}}))$ is the measurement redundancy of the PMU set $S(n_{\text{PMU}})$, which is equal to the number of rows of the Jacobian matrix $\boldsymbol{H}$. The minimum value of (14) is achieved when a PMU set, $S(n_{\text{PMU}})$, provides full network observability $\left(\text{nullity}\begin{pmatrix} \boldsymbol{H} \\ \boldsymbol{C} \end{pmatrix} = 0\right)$ with the minimum number of PMUs distributed around the network with the maximum measurement redundancy.

3.2.4 Optimisation algorithm: The steps of the proposed TS algorithm for the solution of the OPP problem are as follows:

Step 1: Initialise the PMU placement set using the greedy algorithm (Section 3.2.2).
Step 2: Empty the Tabu list (no Tabu-active moves).
Step 3: Evaluate the current solution's value, by computing $Q(n_{\text{PMU}}, S(n_{\text{PMU}}))$. If $Q(n_{\text{PMU}}, S(n_{\text{PMU}})) < w_1$, then remove randomly a PMU from the current PMU set.
Step 4: Create a candidate list of solutions in the neighbourhood of the current solution, $S(n_{\text{PMU}})$. A candidate solution is created by moving a PMU from the installed bus to all the buses where no PMU is placed. Thus, the number of the candidate solutions will be $n_{\text{PMU}}(N - n_{\text{PMU}})$.
Step 5: Evaluate all candidate solutions using (14) and sort them in ascending order.
Step 6: Choose as new current solution, $S'(n_{\text{PMU}})$, that solution from the candidate list, which does not contain Tabu-active moves and has the lowest evaluation value, even if it is worse than the solution of the previous iteration. In this step, aspiration criteria is activated and overrides Tabu list restrictions, in case a candidate solution is better than the best solution found so far, $S_{\text{BSF}}(n_{\text{PMU}})$.
Step 7: Update the Tabu list and set the move that created the new current solution as forbidden for as many iterations as the size of the Tabu list. The size of the Tabu list is called Tabu length (TL); it is constant and depends on the size of the system.
Step 8: If a pre-specified number of TS iterations (TSI) is reached, the algorithm is terminated by returning back the best solution found so far, else move to Step 1.

3.2.5 MTS against RTS: As a result of the stochasticity, which is introduced in the initialisation of the algorithm (Section 3.2.2), as well as in Step 3 of the TS optimisation procedure (Section 3.2.4), the proposed methodology, shown in Fig. 1, may yield different PMU placement sets with the same or different number of PMUs in every execution. Two different approaches are proposed for executing several times the TS algorithm and differentiating the initialisation of the TS algorithm, as described in the following.

In the first approach, called MTS, the procedure shown in Fig. 1 is executed for multiple runs. The initial solution is always provided by the greedy algorithm (Section 3.2.2). At the end of all multiple runs, the algorithm calculates the number of runs for finding the minimum number of PMUs. This rate is called the success rate of the MTS algorithm and it indicates its efficiency.

In the second approach, which is called RTS, the TS algorithm is recursively executed to solve the OPP problem. For a given number of executions, the first execution of the RTS is identical to the one presented in Section 3.2.4. In the rest of the executions, RTS uses as initial solution the best solution found in the previous executions. The success rate of the RTS is calculated with the same way as in MTS.

4 Results and discussion

RTS and MTS methods have been tested on several systems, including the IEEE 14, 30, 57 and 118-bus systems, the NE 39-bus system, as well as the 2383-bus Polish power system [40], so as to investigate the application of the proposed method in large-scale power systems. The basic configuration of all test systems is shown in Table 1. By setting the number of executions equal to 40, reliable results are guaranteed.

4.1 Impact of TS parameters

The parameters of the TS algorithm are: (i) the size of the Tabu list (TL) and (ii) the number of TSI. The correct setting of these parameters is of high importance. Setting a too small value for TL can lead to cycling through a fixed sequence of moves. On the other hand, selecting a large value of TL may often reject promising moves, increasing the computational effort as a result. TSI actually defines the number of cycles that the neighbourhood search lasts and its large value augments the computational time. There is no proven method that determines the optimal values of TS parameters for the solution of the OPP problem; hence the trial and error method is used in all case studies.

Table 2 shows the impact of TS parameters on the efficiency of MTS method for the IEEE 57-bus test system. In this case, the PMU placement initialisation is implemented by the greedy algorithm, whereas the placement of PMU in zero injection nodes is prohibited. It should be noted that the value of TL influences in a higher grade the efficiency of the MTS than the value of TSI. For TSI = 100 and TL = 10 the success rate of the algorithm is 100%, which means that the algorithm provided the minimum number of 11 PMUs in all 40 multiple runs.

Table 3 presents the optimal values of the TS parameters that provide the highest success rates for the case studies of Table 1 with the MTS approach.

Table 2 Impact of TS parameters on the efficiency of the MTS method for the IEEE 57-bus test system

TSI	TL	Minimum number of PMUs	Success rate, %
100	20	11	100
100	10	11	100
100	5	11	38
50	20	11	95
50	10	11	80
50	5	11	25
10	10	11	22.5
10	5	11	25

4.2 Impact of initialisation scheme

TS algorithm is a metaheuristic algorithm that requires an initial solution. In this paper, three schemes of TS initialisation are investigated and their impact on the success rate of the algorithm is examined. The proposed schemes are as follows:

1. The first scheme is applicable for the MTS. This scheme uses the greedy algorithm of Section 3.2.2 that prohibits PMU placement at zero injection nodes.
2. The second scheme, also applicable for the MTS, uses the greedy algorithm of Section 3.2.2 and permits the PMU placement at zero injection nodes. It should be noted that the second scheme provides an initial solution with more PMUs than the first scheme.
3. The third scheme is applicable for the RTS, where the TS is also executed 40 times; in each execution, RTS uses as initial solution the best solution found in the previous executions.

Table 4 shows the impact of the initialisation scheme on the success rate of the proposed TS for the IEEE 57-bus test system. It is concluded that for the same values of TS parameters, the proposed RTS method provides higher success rate than the MTS method; this is the first reason why RTS is proposed for the solution of the OPP problem.

Table 1 Data for the test systems

System	Number of branches	Number of zero injection buses	Locations of zero injection buses
IEEE 14-bus	20	1	7
IEEE 30-bus	41	5	6, 9, 11, 25, 28
NE 39-bus	46	12	1, 2, 5, 6, 9, 10, 11, 13, 14, 17, 19, 22
IEEE 57-bus	78	15	4, 7, 11, 21, 22, 24, 26, 34, 36, 37, 39, 40, 45, 46, 48
IEEE 118-bus	179	10	5, 9, 30, 37, 38, 63, 64, 68, 71, 81
2383-bus	2896	552	–

Table 3 Optimal values of TS parameters, minimum number of PMUs and success rate for the MTS method (greedy algorithm prohibiting location of PMUs at zero injection buses)

Test system	Optimal values of TS parameters		Minimum number of PMUs	Success rate, %
	TSI	TL		
IEEE 14-bus	10	2	3	100
IEEE 30-bus	2	1	7	100
NE 39-bus	20	10	8	92.5
IEEE 57-bus	100	10	11	100
IEEE 118-bus	200	50	28	25
2383-bus	400	100	553	20

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Table 4 Impact of initialisation scheme and TS parameters on the success rate of the TS method for the IEEE 57-bus system

Optimal values of TS parameters		Minimum number of PMUs	Success rate, %		
TSI	TL		First initialisation scheme: MTS using greedy algorithm prohibiting location of PMUs at zero injection buses	Second initialisation scheme: MTS using greedy algorithm permitting location of PMUs at zero injection buses	Third initialisation scheme: RTS
100	20	11	100	97.5	100
100	10	11	100	100	100
100	5	11	38	37.5	100
50	20	11	95	90	100
50	10	11	80	72.5	85
10	10	11	22.5	12.5	100
10	5	11	25	7.5	90

Especially in MTS, the first initialisation scheme provides higher success rate than the second one, for the same values of TS parameters. Furthermore, RTS needs a smaller TSI value than MTS in order to reach high success rates and the smaller the TSI value, the smaller is the computational effort (Section 4.1); this is the second reason why RTS is proposed for the solution of the OPP problem. Fig. 2 shows the behaviour of the three

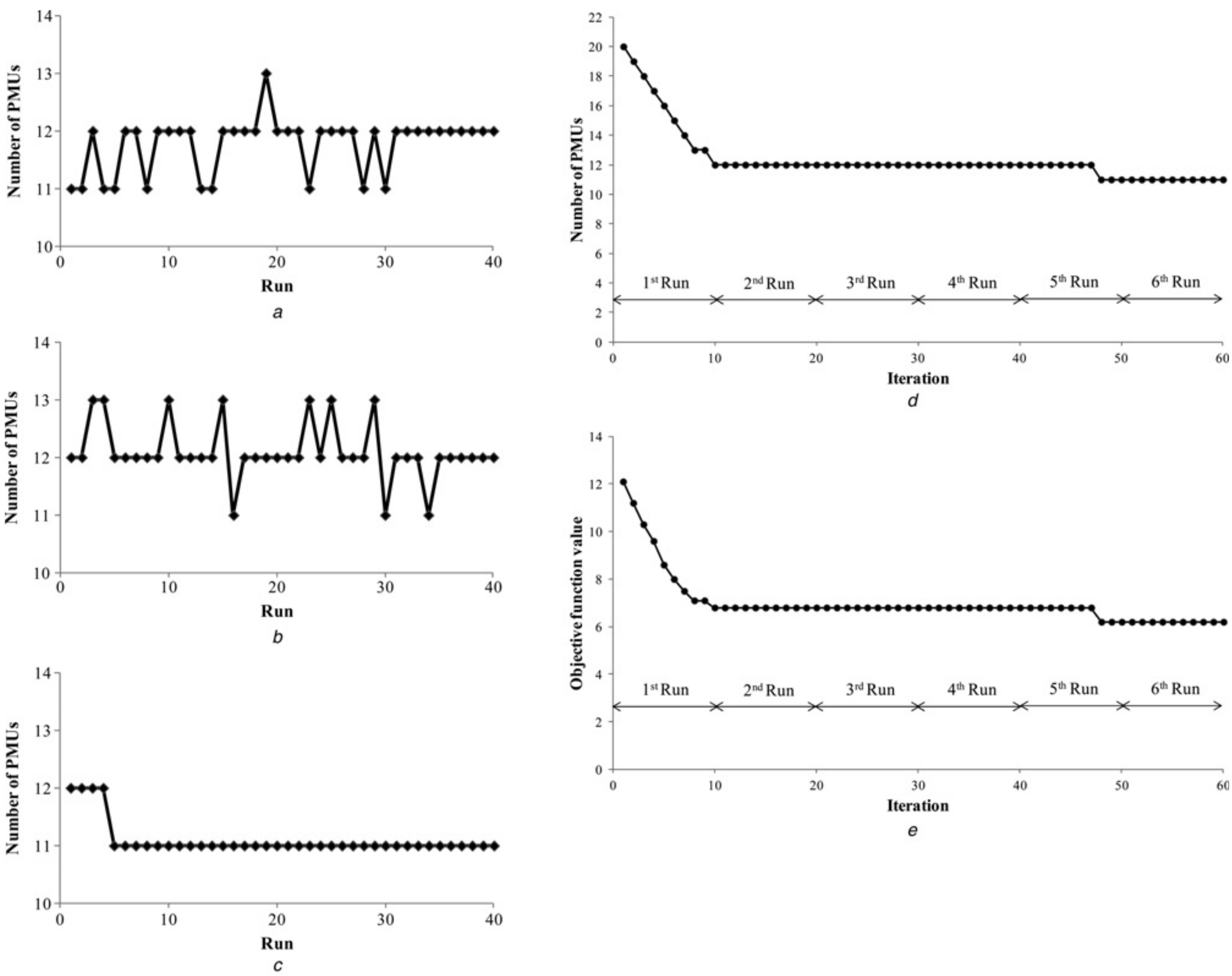

Fig. 2 *Impact of initialisation scheme on IEEE 57-bus test system using TSI = 10 and TL = 5*

a Number of PMUs for the first initialisation scheme
b Number of PMUs for the second initialisation scheme
c Number of PMUs for the third initialisation scheme
d Number of PMUs for the third initialisation scheme and the first six runs, where each of run is composed of ten iterations
e Value of objective function for the third initialisation scheme and the first six runs

initialisation schemes for TSI = 10 and TL = 5, for the IEEE 57-bus test system. It can be seen from Fig. 2*c* that the optimal solution has been found within the first five executions of RTS; this is the third reason why RTS is proposed for the solution of the OPP problem. The variability in the minimum number of PMUs shown in Figs. 2*a* and *b* is due to the fact that in MTS, the solution in each execution is independent from the solution of the previous execution. On the other hand, in RTS, due to its initialisation scheme, the number of PMUs in each execution will be equal or smaller than the number of PMUs of the previous execution, as Fig. 2*c* confirms.

Fig. 2*d* depicts the number of PMUs for the third initialisation scheme presenting only the first six runs, where each of the six runs is composed of ten iterations; in fact Fig. 2*d* presents the first six runs of Fig. 2*c* showing the evolution of the algorithm on an iteration-by-iteration basis. Similarly, Fig. 2*e* shows, for the first six runs of the third initialisation scheme, the evolution of the value of the objective function (14), the weights of which for the RTS method are set as follows: $w_1 = 57$, $w_2 = 1$ and $w_3 = 0.1$, while TSI = 10 and TL = 5. In the first iteration of the first run, the initial solution computed by the greedy algorithm implies a placement with 20 PMUs (Fig. 2*d*) and the objective function value for that placement is 12.1 (Fig. 2*e*). At the end of the first run, that is, at the 10th iteration, the RTS algorithm results in a placement with 12 PMUs (Fig. 2*d*) and the objective function value is 6.8 (Fig. 2*e*). In the second run, that is, iterations 11–20, the solution of the 10th iteration (best solution of the first run) is used as the initial solution for the 11th iteration. After five runs, and more specifically in the 48th iteration, the optimal solution found corresponds to 11 PMUs and has an objective function value of 6.2. Consequently, five runs, and more specifically 48 iterations, were enough for the RTS to find the best solution.

Table 5 Optimal PMU locations obtained by the first initialisation scheme (MTS using greedy algorithm prohibiting location of PMUs at zero injection buses)

System	Minimum number of PMUs	PMU location (bus #)
IEEE 14-bus	3	2, 6, 9
IEEE 30-bus	7	2, 3, 10, 12, 18, 24, 27
		2, 3, 10, 12, 19, 24, 27
		3, 7, 10, 12, 18, 24, 27
		2, 4, 10, 12, 18, 24, 27
		2, 4, 10, 12, 19, 24, 27
NE 39-bus	8	3, 8, 13, 16, 20, 23, 25, 29
		3, 8, 10, 16, 20, 23, 25, 29
		3, 8, 16, 20, 23, 29, 32, 37
		3, 8, 12, 16, 20, 23, 25, 29
IEEE 57-bus	11	1, 4, 13, 19, 25, 29, 32, 38, 41, 51, 54
		1, 4, 13, 20, 25, 29, 32, 38, 51, 54, 56
		1, 6, 13, 19, 25, 29, 32, 38, 51, 54, 56
		1, 6, 13, 19, 25, 29, 32, 38, 41, 51, 54
IEEE 118-bus	28	3, 8, 11, 12, 17, 20, 23, 29, 34, 37, 40, 45, 49, 53, 56, 62, 73, 75, 77, 80, 85, 86, 91, 94, 101, 105, 110, 115
		3, 8, 11, 12, 19, 22, 27, 31, 32, 34, 37, 40, 45, 49, 53, 56, 62, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110
		3, 8, 11, 12, 17, 20, 23, 29, 34, 37, 40, 45, 49, 52, 56, 62, 71, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110, 115
		3, 8, 11, 12, 19, 21, 27, 31, 32, 34, 37, 42, 45, 49, 52, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110

The OPP sets for all case studies obtained by the first, the second and the third initialisation schemes are presented in Tables 5–7 respectively. It should be noted that the MTS method yields more than one PMU placement sets with the same minimum number of PMUs, while the RTS method yields only one. Table 7 shows that the CPU time spent by the proposed method is reasonable even for the 2383-bus test system, and this makes the proposed method computationally very attractive to effectively solve the OPP problem for large-scale power systems. Owing to space limitations, the locations of zero injection buses and PMUs are not provided in Tables 1 and 7 respectively.

4.3 Comparison with other methods

To investigate the minimum number of PMUs (n_{PMU}) and the measurement redundancy provided by the proposed RTS method and compare the results with other methods, two measurement redundancy indices are introduced: (i) the total number of channels per PMU (λ), and (ii) the total number of channels (TNC). Index TNC is equal to the sum of PMU channels corresponding to the voltage and the current phasor measurements provided by PMUs;

Table 6 Optimal PMU locations obtained by the second initialisation scheme (MTS using greedy algorithm permitting location of PMUs at zero injection buses)

System	Minimum number of PMUs	PMU location (bus #)
IEEE 14-bus	3	2, 6, 9
IEEE 30-bus	7	1, 5, 10, 12, 18, 23, 27
		1, 5, 10, 12, 18, 24, 27
		3, 5, 10, 12, 18, 23, 27
		3, 5, 10, 12, 18, 24, 27
		1, 5, 10, 12, 19, 24, 27
		2, 4, 10, 12, 18, 24, 27
		1, 2, 10, 12, 18, 24, 27
NE 39-bus	8	3, 8, 13, 16, 20, 23, 25, 29
IEEE 57-bus	11	1, 6, 13, 19, 25, 29, 32, 38, 41, 51, 54
		1, 4, 13, 20, 25, 29, 32, 38, 51, 54, 56
		1, 4, 13, 19, 25, 29, 32, 38, 41, 51, 54
		1, 4, 13, 20, 25, 29, 32, 38, 41, 51, 54
IEEE 118-bus	28	3, 8, 11, 12, 17, 21, 27, 31, 32, 34, 37, 40, 45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110
		3, 8, 11, 12, 19, 22, 27, 31, 32, 34, 37, 40, 45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110

Table 7 Optimal PMU locations and average CPU time obtained by the third initialisation scheme (RTS)

System	Minimum number of PMUs	PMU location (bus #)	CPU time, s
IEEE 14-bus	3	2, 6, 9	0.01
IEEE 30-bus	7	1, 2, 10, 12, 18, 24, 27	0.33
NE 39-bus	8	3, 8, 10, 16, 20, 23, 25, 29	0.57
IEEE 57-bus	11	1, 6, 13, 19, 25, 29, 32, 38, 41, 51, 54	2.11
IEEE 118-bus	28	3, 8, 11, 12, 17, 21, 27, 31, 32, 34, 37, 40, 45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110	29.12
2383-bus	553	–	917.36

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Table 8 Comparison of the proposed RTS and other OPP methods (abbreviations used: TS = Tabu search; GA = genetic algorithm; SA = simulated annealing; IGA = immune GA; IP = integer programming; ILP = integer linear programming)

System	IEEE 14-bus			IEEE 30-bus			NE 39-bus			IEEE 57-bus			IEEE 118-bus			2383-bus		
Method	n_{PMU}	λ	TNC	n_{PMU}	λ	TNC	n_{PMU}	λ	TNC	n_{PMU}	λ	TNC	n_{PMU}	λ	TNC	n_{PMU}	λ	TNC
Proposed RTS	3	5.00	15	7	4.71	33	8	4.37	35	11	4.36	48	28	5.25	147	553	4.33	2344
TS [32]	3	5.00	15	–	–	–	10	4.30	43	13	4.15	54	–	–	–	–	–	–
GA [35]	3	5.00	15	7	4.14	29	–	–	–	12	–	–	29	–	–	–	–	–
SA [31]	3	5.00	15	7	4.57	32	–	–	–	13	4.23	54	–	–	–	–	–	–
IGA [34]	3	5.00	15	7	4.14	29	–	–	–	11	4.36	48	29	4.79	139	–	–	–
IP [27]	3	5.00	15	7	4.29	30	–	–	–	13	4.31	56	29	5.10	148	–	–	–
ILP [28]	3	5.00	15	7	4.43	31	8	3.88	31	11	4.36	23	28	4.93	138	553	–	–
ILP [29]	–	–	–	–	–	–	–	–	–	–	–	–	28	5.00	140	–	–	–

consequently, TNC varies according to the minimum number and sites of PMUs provided by each method. A more secure criterion to compare the measurement redundancy of the proposed algorithm with other techniques is to use the index λ (total number of channels per PMU), which is defined as

$$\lambda = \frac{\text{TNC}}{n_{\text{PMU}}} \tag{15}$$

Table 8 compares the results of the proposed RTS method with those of some methods from the literature. As can be seen, the proposed RTS method finds the best results concerning the minimum number of PMUs (n_{PMU}) and the maximum measurement redundancy (λ), for all cases. On the other hand, other methods find either the same minimum number of PMUs or higher. It should be noted that the redundancy indicator (λ) for the proposed method is the highest in all cases, even if the total number of channels (TNC) is not always the highest, because of the different minimum number of PMUs (n_{PMU}) found by the proposed method in comparison with the other techniques.

5 Conclusion

This paper introduces a RTS method to obtain a solution for the OPP problem by checking network observability with a numerical algorithm. The impact of three different TS initialisation schemes, together with the two TS parameters (TL and TSI), on the optimal solution is investigated. The experimental results verify the superiority of the proposed RTS method over the MTS method. The effectiveness and flexibility of the proposed scheme is demonstrated by the simulation results tested on the four IEEE test systems, the NE 39-bus system and the 2383-bus Polish system. The proposed RTS algorithm determines the minimum number of PMUs, unlike other methods which find either the same minimum number of PMUs or even higher.

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